NUMERICAL CALCULATION OF SNOW AVALANCHE RUNOUT DISTANCES

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ABSTRACT

Snow avalanches threaten villages, roads and railways in most mountainous regions in the world. The extreme avalanche winters of 1951 and 1999 in Switzerland clearly demonstrated the importance of avalanche dynamics calculations, in preparing hazard maps for snow avalanches. Today's hazard mitigation strategies rely both on practical experience and avalanche dynamics models that predict snow avalanche descent paths, runout distances and impact pressures. This paper discusses the practical difficulties of applying a multi-dimensional numerical analysis tool to predict snow avalanche runout distances. The tool is embedded in a Geographical Information System (GIS) to simplify the specification of input and to help the interpretation of numerical results. Essential for an accurate prediction of avalanche runout is not only the flow friction, but also the specification of the initial release conditions – the fracture height and area. These are semi-automatically generated with the help of the GIS system. An advanced TVD numerical finite difference scheme then solves the differential equations that govern dense snow avalanche flow in general terrain. The primary results are pressure maps that define runout distances and the extent of avalanche danger. Two example calculations are presented. The first is an application of a single avalanche track where different combinations of release zones are discussed. The second example analyses the usefulness of protective forests to reduce avalanche runout distances over a large area.

KEY WORDS

Hazard mitigation, snow avalanches, numerical modeling, GIS.

INTRODUCTION

In Switzerland snow avalanches, debris flows and rockfalls pose a serious threat to mountain communities. Hazard mitigation involves technical (e.g. supporting structures in avalanche starting zones), organizational (avalanche warning, road closings) and planning (hazard

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zoning) measures. Central to the latter category is the concept of a hazard map, which zones land into different danger categories depending on avalanche activity and impact pressure. Avalanche dynamics calculations are used to predict an extreme avalanche event and to delineate the different hazard zones (Gruber et al. 1998). The analytical Voellmy-Salm model (Salm et al. 1990) and, since 1999, the one-dimensional avalanche dynamics program AVAL-1D (Christen et al. 2002) have been successfully applied in practice for this purpose all over the world. However, problems like complicated topographies and release areas, strange runout behavior (avalanche arms) and deflecting or catching dams require more sophisticated avalanche dynamic models that are able to describe avalanche dynamics problems in a multi-dimensional terrain.

For this reason we developed a numerical simulation tool that predicts snow avalanche runout distances, flow velocities and impact pressures in general three-dimensional terrain. The used dense-snow-avalanche flow law is a well calibrated, hydraulics-based, depth-averaged continuum model and divides avalanche flow resistance into a dry Coulomb-type friction and a viscous resistance which varies with the square of the flow velocity. An advanced TVD finite difference scheme solves the shallow water equations in general three-dimensional terrain. We then coupled the model with a GIS information system to simplify the specification of terrain and initial conditions. At the end of the paper two example calculations present the practical application of the tool.

GOVERNING DIFFERENTIAL EQUATIONS

In this section we present the governing differential equations for dense snow avalanches. The model is a generalization of the quasi one-dimensional model discussed in detail in (Bartelt et al. 1997, Bartelt et al. 1999). We treat the avalanche as an unsteady flow on a two-dimensional manifold (the mountain) with the flow velocity given by a two-dimensional vector field $\vec{u} = (u,v)$. The flow height h of the avalanche, measured perpendicular to the mountain profile, is the final unknown. We consider only dense flowing avalanches; that is, heavy dense flows of constant density. The governing differential equations of mass and momentum conservation are:

(1) $$\frac{\partial h}{\partial t} + \vec{\nabla}(h\vec{u}) = 0$$

and

(2) $$\frac{\partial h\vec{u}}{\partial t} + \vec{\nabla}(h\vec{u} \otimes \vec{u}) = \vec{f} - \frac{1}{2}\vec{\nabla}\, l h^2 .$$

The vector $\vec{f}$ defines the difference between the gravitational acceleration and friction. The parameter l defines the active-passive flow pressure due to longitudinal straining of the flow body (Bartelt et al. 1999). In principle, the governing equations are similar to the shallow water equations, with the exception of the active-passive pressure. By defining a vector of conservative variables $\vec{U} = (h, h\vec{u})^T$, the system of equations (1 and 2) can be concisely written

(3)
$$\frac{\partial \vec{U}}{\partial t}+\frac{\partial \vec{F}_1}{\partial x_1}+\frac{\partial \vec{F}_2}{\partial x_2}=\vec{G}$$

where x_1 and x_2 define the coordinate system of the manifold. The vectors $\vec{F}_1$ and $\vec{F}_2$ are

(4)
$$\vec{F}_1=\begin{pmatrix} hu \\ hu^2+\frac{1}{2}lh^2 \\ huv \end{pmatrix}$$

and

(5)
$$\vec{F}_2=\begin{pmatrix} hv \\ huv \\ hv^2+\frac{1}{2}lh^2 \end{pmatrix}.$$

The vector $\vec{G}=(0, f_1, f_2)^T$ contains the difference between the gravitational driving force and friction in both coordinate directions. Typically, f consists of a dry Coulomb friction (dependent on the flow height h) and an additional contribution proportional to the velocity squared which takes into account the influence of snow entrainment,

(6)
$$f_1=g_1-fn_u \text{ and } f_2=g_2-fn_v\text{, where } f=mg_3+\frac{g_3\left(u^2+v^2\right)}{xh}.$$

g_1, g_2 and g_3 are the components of the gravitational acceleration in the coordinate directions. The friction parameters m and x define the amount of dry Coulomb friction and the velocity-dependent contribution. These are usually specified according to the Swiss Guidelines for avalanche runout calculation (Salm et al. 1990). The friction parameters m and x are dependent on the avalanche volume.

Except for the right-hand-side vector $\vec{G}$, these model equations are similar to the Euler equations for a two-dimensional isentropic gas, or the two-dimensional shallow water equations. This conservative formulation is best suited for the discretization of the governing equations since it leads to shock capturing discretization methods. In order to study the properties of the system, one uses the quasi-linear formulation obtained by applying the chain rule to (3) and by assuming the property $\vec{\nabla} l=0$,

(7)
$$\frac{\partial \vec{U}}{\partial t}+A\frac{\partial \vec{U}}{\partial x_1}+B\frac{\partial \vec{U}}{\partial x_2}=\vec{G},$$

with

(8)
$$A=\frac{\partial \vec{F}_1}{\partial \vec{U}}=\begin{pmatrix} 0 & 1 & 0 \\ -u^2+lh & 2u & 0 \\ -uv & v & u \end{pmatrix}$$

and

(9) $$B = \frac{\partial \vec{F}}{\partial \vec{U}} = \begin{pmatrix} 0 & 0 & 1 \\ -uv & v & u \\ -v^2 + lh & 0 & 2v \end{pmatrix}.$$

The quasi-linear system (7) is hyperbolic with respect to the time variable, if any real linear combination of the matrices A and B has real eigenvalues. Let $a^2 + b^2 = 1$ then the matrix $aA + bB$ has the spectral decomposition $(aA + bB) = S\Lambda S^{-1}$ with matrices

(10) $$\Lambda = diag\left(au + bv, au + bv + \sqrt{lh}, au + bv - \sqrt{lh}\right),$$

(11) $$S = \begin{pmatrix} 0 & 1 & 1 \\ -b\sqrt{lh} & u + a\sqrt{lh} & u - a\sqrt{lh} \\ a\sqrt{lh} & v + b\sqrt{lh} & v - b\sqrt{lh} \end{pmatrix}$$

and

(12) $$S^{-1} = \frac{1}{2\sqrt{lh}} \begin{pmatrix} 2bu - 2av & -2b & 2a \\ -au - bv + \sqrt{lh} & a & b \\ +au + bv + \sqrt{lh} & -a & -b \end{pmatrix}.$$

The diagonal matrix Λ defines the real eigenvalues showing that the system is hyperbolic; the columns of S are the (right) eigenvectors (Sartoris et al. 2000).

COMPUTER IMPLEMENTATION

At the center of the computer environment is a GIS-based graphical user interface GUI that connects all the different elements of the avalanche dynamics calculations process. Topographical data preparation, input specification and hazard mapping tools are handled by ARC/Info. The numerical simulation model is a C-program and directly connected to the GUI, where the numerical input is generated and all simulation results can be displayed.

GIS

A good digital representation of the topography is crucial for the accuracy of the model results. At present, in Switzerland a Digital Elevation Model (DEM) exists in a raster format with a spatial resolution of 25m (Swisstopo 2001). In the region of Davos, an almost complete database of avalanche events over the last 50 years exists. Using Geographic Information System (GIS) technologies in combination with a DEM, the recorded avalanche release areas were analyzed with respect to topographic characteristics. The statistical analysis resulted in probability distributions for release area extents as a function of the frequency and the topographic parameters (Maggioni et al. 2002, Maggioni et al. 2003).

The resolution of 25m is sufficiently accurate for open slope terrain, but not in steep gullies. Since most of the avalanche paths are at least partially in narrow terrain, there is a strong demand for improving the quality of the DEM in these areas. Therefore GIS-functionalities have been implemented to improve the accuracy of the DEM in steep gullies (Gruber 2001).

INPUT PARAMETERS

The input to the numerical model can be divided into the following three categories:

Initial Conditions

A semi-automatic procedure to define potential avalanche release areas is applied. Generally, avalanches can release on slopes between a lower slope boundary (LSB, normally: about 30°) and an upper slope boundary (USB, normally: about 50°) outside densely forested areas. Within this slope range, ridges can divide one single steep slope into many potential release zones, that normally do not release simultaneously. For avalanche hazard mapping purposes, only large avalanches have to be taken into account. Small release areas can therefore be neglected. To extract potential release areas based on the above-mentioned criteria, an AML was written. As input, the DEM and a grid containing the forested areas (extract of a digital topographic map with a scale 1:25'000) is used.

In a first step, the non-forested areas with a slope angle between LSB and USB are extracted. In a second step, the size and lengths of these areas are analyzed. Areas below a specific minimum area value (MAV) or a minimum release downhill length (MRDL) were eliminated.

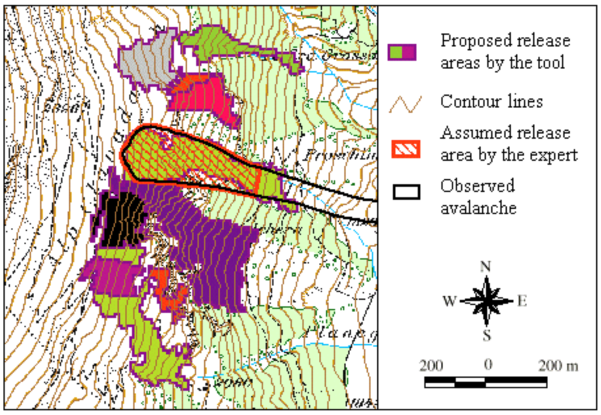

Figure 1: Result of the semi-automatic classification of release areas with a parameter set for extreme avalanche events (Digital Elevation Model and Digital Map:© Swiss Federal Office of Topography)

In a third step, the decision is made whether ridges are dividing a potential release zone into two or more single release zones. This decision is governed by a distance threshold value (DTV). This value defines the maximum distance between two neighboring lower boundary cells. If the distance is greater than the specified DTV, they are considered not to belong to the same potential release area.

Of course, the values for every terrain and vegetation class have to be carefully chosen by an avalanche expert. As for the semi-automatic procedure to delineate the release areas (see

Figure 1), also within this procedure the avalanche expert has the opportunity of interactively changing the friction parameter values in areas where he does not agree with the automatic classification procedure (Gruber 2001).

Track Specifications

The model does not require any track or width specification, since both elements are calculated. Only the calculation domain area must be specified, i.e. the maximum area that is potentially influenced by the avalanche. The smaller the domain area, the faster the two-dimensional simulation is completed.

Friction Parameters

The friction parameters µ (dry-friction) and ? (turbulent or viscous friction) depend on the snow properties, the initial release volume of the avalanche, the vegetation coverage and on the confinement of the topography. GIS-functionalities are used to define the initial release volume, to vary the snow properties according to the altitude (i.e. it is assumed that in higher regions the snow is generally more loose), to take into account spatially varying vegetation (mainly: forested areas) and to classify the DEM into different areas of confinement. For the latter procedure, the GRID-function "curvature" is used to define the curvature perpendicular to the slope direction. All these spatial classifications were performed so that the friction parameters can be varied along the avalanche track.

OUTPUT RESULTS

The numerical simulation model provides information about flow and deposition depths, velocities and impact pressures (see Figure 2). For the purpose of avalanche hazard mapping, the impact pressure is the most important result since the hazard zones are mainly defined by it. GIS allows the visualization of the pressure forces on maps as well as the combination of results of varying scenarios (i.e. return periods or different release area assumptions) into a combined pressure zone map. This pressure zone map is an important base for the final delineation of a hazard map, which is legally binding.

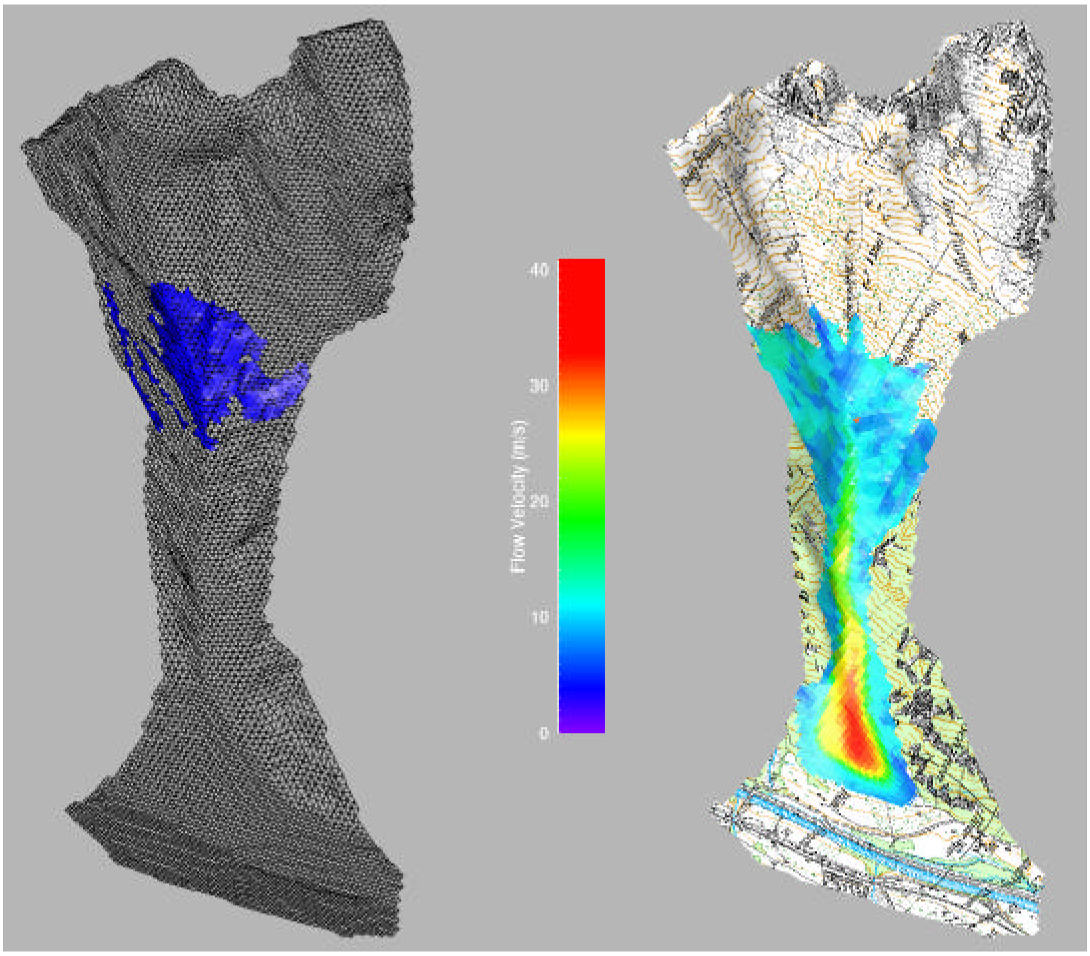

Figure 2: An example of a snow avalanche simulation. The picture on the left shows the finite difference mesh with the initial conditions, a snow slab with a fracture height of 1m. The simulation domain contains 11'000 triangles. The avalanche enters the runout zone with a velocity of over 40 ms^{-1} and laterally spreads. The legend in the middle depicts the flow velocities. The highway at the valley floor is threatened. The calculation took less than 10 minutes on a Windows PC.

EXAMPLES

In this section, we present two model applications. The first example demonstrates the power of the model for avalanche hazard mapping purposes in a single, but complex avalanche track, named "Tallawine" that endangers the settlement of Klosters, Canton GR, Switzerland. The second example demonstrate the usefulness of the tool with respect to large area avalanche hazard mapping in Davos, Canton GR, Switzerland.

KLOSTERS

The potential release area of the "Tallawine" (Figure 3, left) is large and structured into several sub-basins. The GIS-part of our simulation tool provides very useful help in determining the potential release areas and its subdivision into several sub-basins.

Two scenarios have been calculated: The first scenario assumes that all potential fracture areas will release at the same moment. The second scenario was that all of the four sub-basins will release separately. In Figure 3, the results of Scenario 1 are shown on the map in the middle, and the maximum extent of Scenario 2 is shown on the map on the right side. For comparison, the extent of the official hazard map is shown as red and blue lines. The runout distance of Scenario 1 is larger than the official hazard zone and the Scenario 2 is smaller than the official hazard zone. This is in good agreement with the basic assumptions for the official hazard map, namely that more than one, but not all sub-basins of the potential release area will be triggered at once.

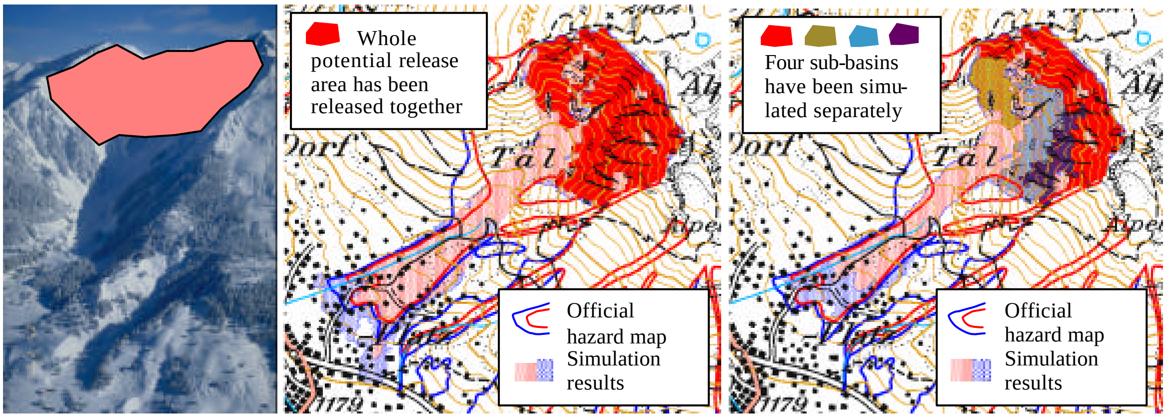

Figure 3: Potential release area of the "Tallawine" Klosters (left); results of Scenario 1 (middle) and results of Scenario 2 (right)

DAVOS

In Switzerland, the maintenance of forest areas that protect settlements against avalanches is subsidized by the federal government. In order to distribute the money among all settlements of Switzerland appropriately, there is a need for a objective determination which forest has a protective function. Our GIS-tool in combination with the numerical avalanche model provides such an objective tool and is applicable over very large areas (> 10'000 km^2). The method is to determine automatically all potential release areas that are located in forested area, but removing the effect of the forest for the purpose of the calculation. These potential release areas are then divided according to their potential avalanche release volume into large, medium and small release areas, since the friction parameters are volume dependent. The release areas of each size class are then calculated separately to determine the runout distance. In the case when the additional hazard clearly endangers settlements, roads or buildings, the forest is considered to have a protective function. In the case when no damage potential is present, no protective function is assigned.

In Figure 4, two scenarios are depicted: On the left side, a simulation of the actual avalanche hazard is shown. On the right side of Figure 4, a worst-case scenario is simulated with the assumption that no forest exists. In the areas of the red circles, the additional hazard would clearly endanger settlements and subsequently, the forest areas have to be considered as protective forests.

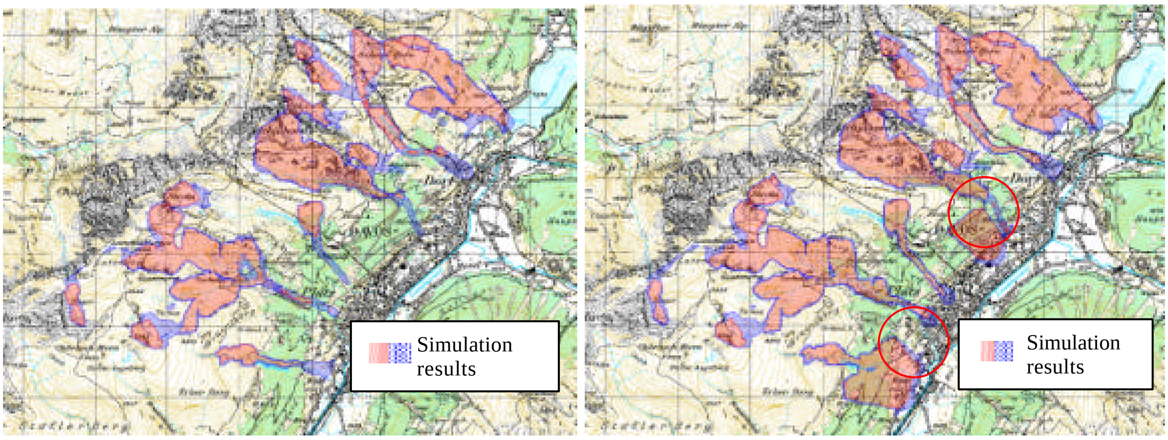

Figure 4: Simulation of the actual avalanche hazard in Davos (left); worst-case simulation with all potential release areas within today's existing forests included (right)

Several Cantons have already defined the extent of their protective forests by different means. Therefore, it is interesting to compare the results of this almost automatic and therefore objective tool to the existing protective forest delineations. In Figure 5, the simulation results (red/blue) are compared to the protective forests delineated by the authorities of the Canton of Valais. The green/red-dotted areas show the intersections of both methods. The light green areas are the once, that are only assigned by the Canton Valais, but not by our simulation. In most cases shown in Figure 5, the light green areas are simply too flat for avalanches or other natural hazards.

With the help of this tool it is possible to assign the protective forest of all Cantons in Switzerland with a high precision and in an objective way. This allows us to overcome the existing differences in order to distribute the subsides appropriately among all Cantons of Switzerland.

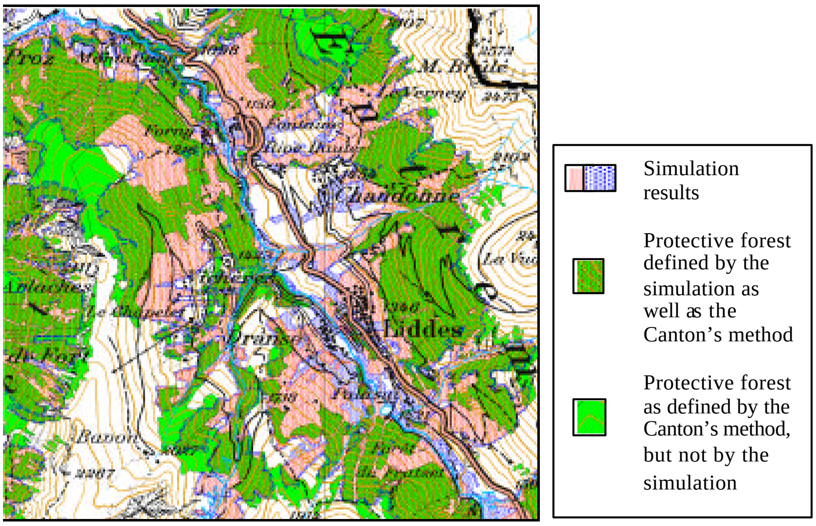

Figure 5: Simulation results (red/blue) are compared to the protective forests delineated by the authorities of the Canton of Valais

CONCLUSIONS AND OUTLOOK

We have shown that the combination of numerical simulation tools and GIS can be helpful in solving practical avalanche problems. Runout distances can be calculated in three-dimensional terrain; comparisons with well-tested hazard maps and actual avalanche observations show a good agreement. Although these results were obtained with semi-automatic methods to generate initial conditions and track characteristics, in-depth knowledge of the process (snow avalanches) is still required in order to achieve reasonable results, especially in complex situations. GIS-based systems and numerical simulation tools should not and cannot be used as a black box.

In future, we plan to transfer the methodologies that we have developed for snow avalanches to other natural hazards such as debris flows and rockfalls. However, the transfer must take into account the particularities of the different processes. For example, although the simulation tool can be used to study debris flows, the specification of the initial conditions is completely different. The starting conditions for snow avalanches are relatively simple – a fracture slab of some height and area. Starting conditions for debris flows are complex and must take into account precipitation and other geological features. This problem also exists for rockfalls, where the location, size and shape of a rock must be defined. Thus, in order that numerical simulation tools solve real problems, the physics of the processes must always be understood.

To improve the GIS user interface it is planned to improve the visualization of the numerical simulation results by creating 3-D perspective views of avalanche tracks with draped ortho-photos or maps as background and to animate the avalanche simulations on these views. This improvement would be very useful for presenting the results of numerical simulation models to an endangered community, in order to increase their understanding of the avalanche hazard on a specific slope.

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